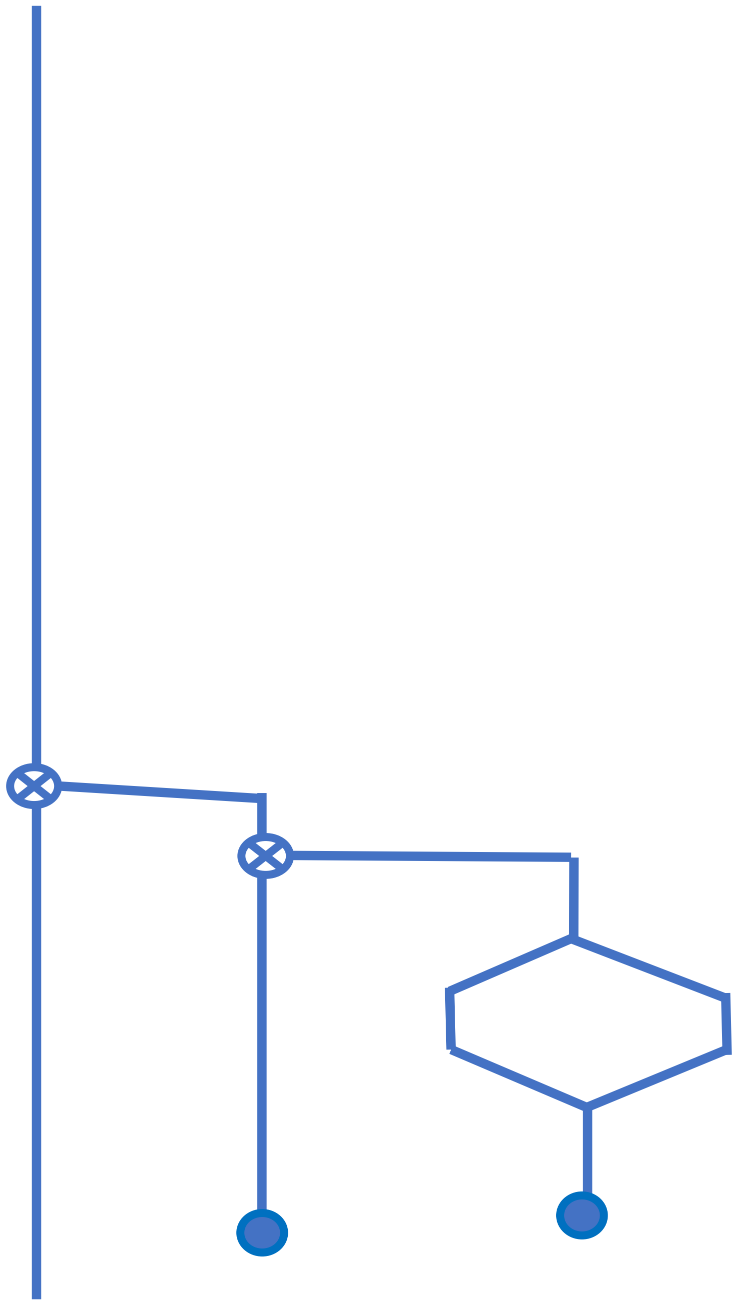


# A Clever Computation of Pi

SVFIG

Feb. 24, 2024

Bill Ragsdale

# How To Compute $\pi$?

$$\pi = 2\left(1 + \frac{1}{3} + \frac{1\cdot 2}{3\cdot 5} + \frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7} + \frac{1\cdot 2\cdot 3\cdot 4}{3\cdot 5\cdot 7\cdot 9} + \frac{1\cdot 2\cdot 3\cdot 4\cdot 5}{3\cdot 5\cdot 7\cdot 9\cdot 11} + \cdots\right)$$

That is a lot of work.

I want the answer in one step.

# The Logic

I only want one term/variable.

$$\pi = N + \mathrm{f(N)}$$

Where f(N) is the deviation from $\pi$.

## The Logic

$$\pi = N + \mathrm{f}(\mathrm{N})$$

But, we do not know N, so we have to guess.

# The Logic

$$\pi = N + \mathrm{f(N)}$$

But, we do not know N,
so we have to guess.

$$\pi \approx N_1 = N_0 + \mathrm{f}(\mathrm{N}_0)$$

And repeat iteratively.

# The Logic

$$\pi \approx N_1 = N_0 + \mathrm{f}(\mathrm{N}_0)$$

$$\pi - N_0 \approx \mathrm{f}(\mathrm{N}_0), \text{ the error}$$

# The Logic

$$\pi \approx N_1 = N_0 + \mathrm{f}(\mathrm{N}_0)$$

$$\pi - N_0 \approx \mathrm{f}(\mathrm{N}_0)\text{, the error}$$

Thus, as $\mathrm{f}(\mathrm{N}_0) \to 0$

Then $N_0 \to \pi$

# The Logic

Thus, as $f(N_0) \rightarrow 0$

Then $N_0 \rightarrow \pi$

So, what $f(\pi) = 0$?

## The Logic

$$N^2 = \pi^2$$

$$\ln(\mathrm{N}) = \ln(\pi)$$

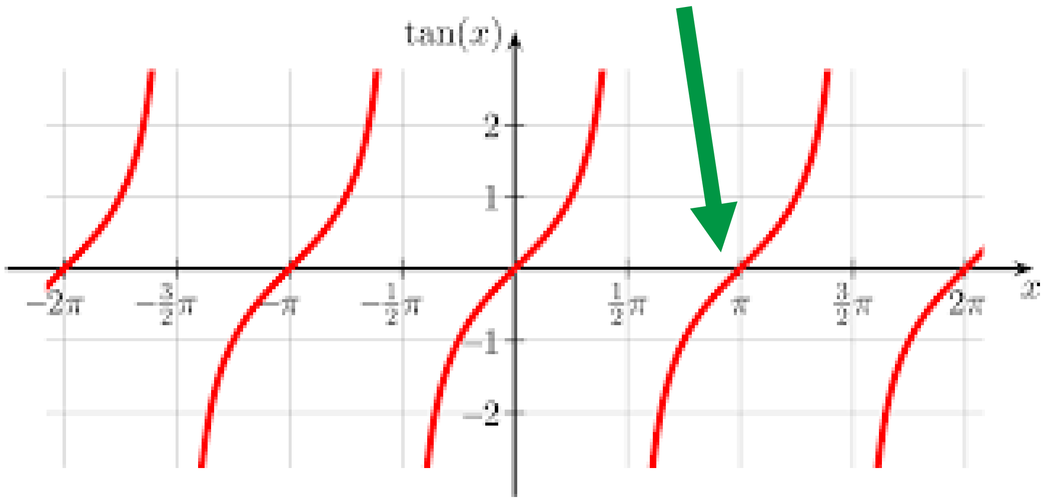
$$\mathrm{e}^{\mathrm{N}} = \mathrm{e}^{\pi}$$

$$But\ these\ are\ not\ zero\ for\ \pi.$$

# The Logic

We need a cyclic function.

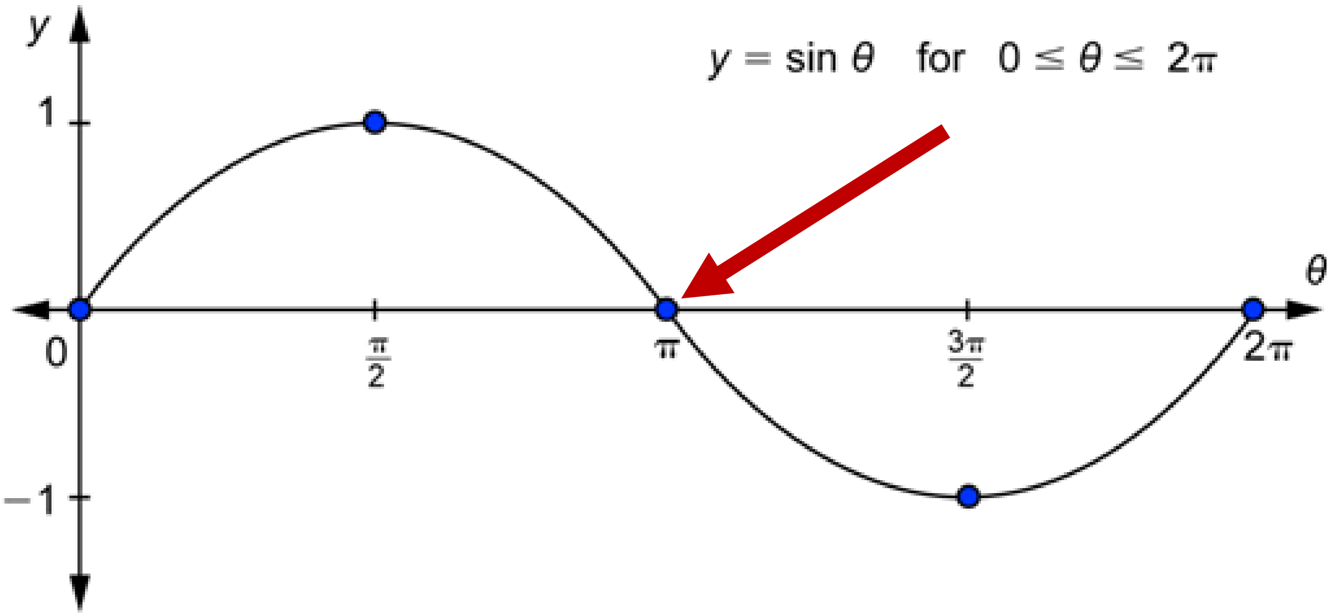
How about tangent?




Zero at π but it is not continuous

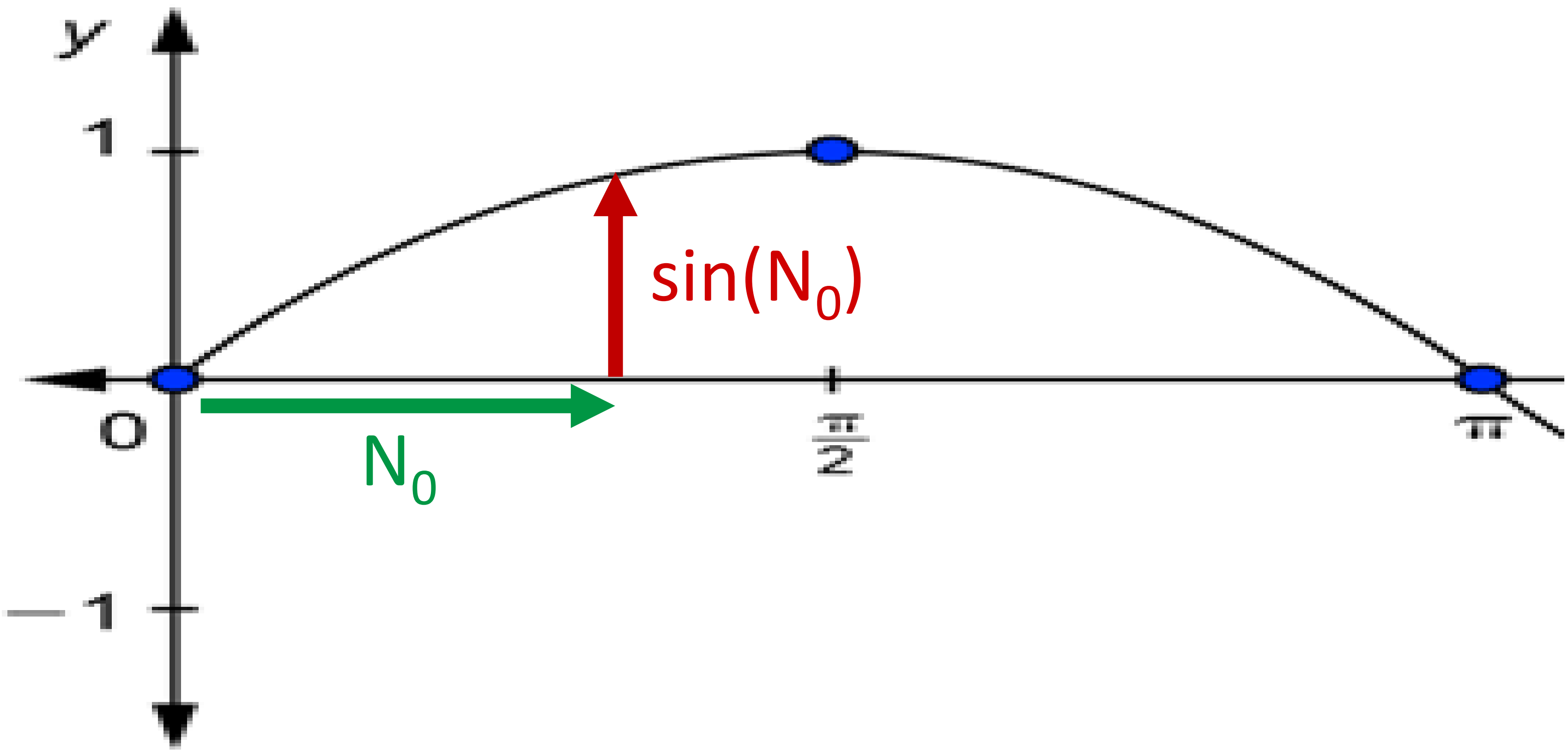
Ah Ha!

# How about sine?

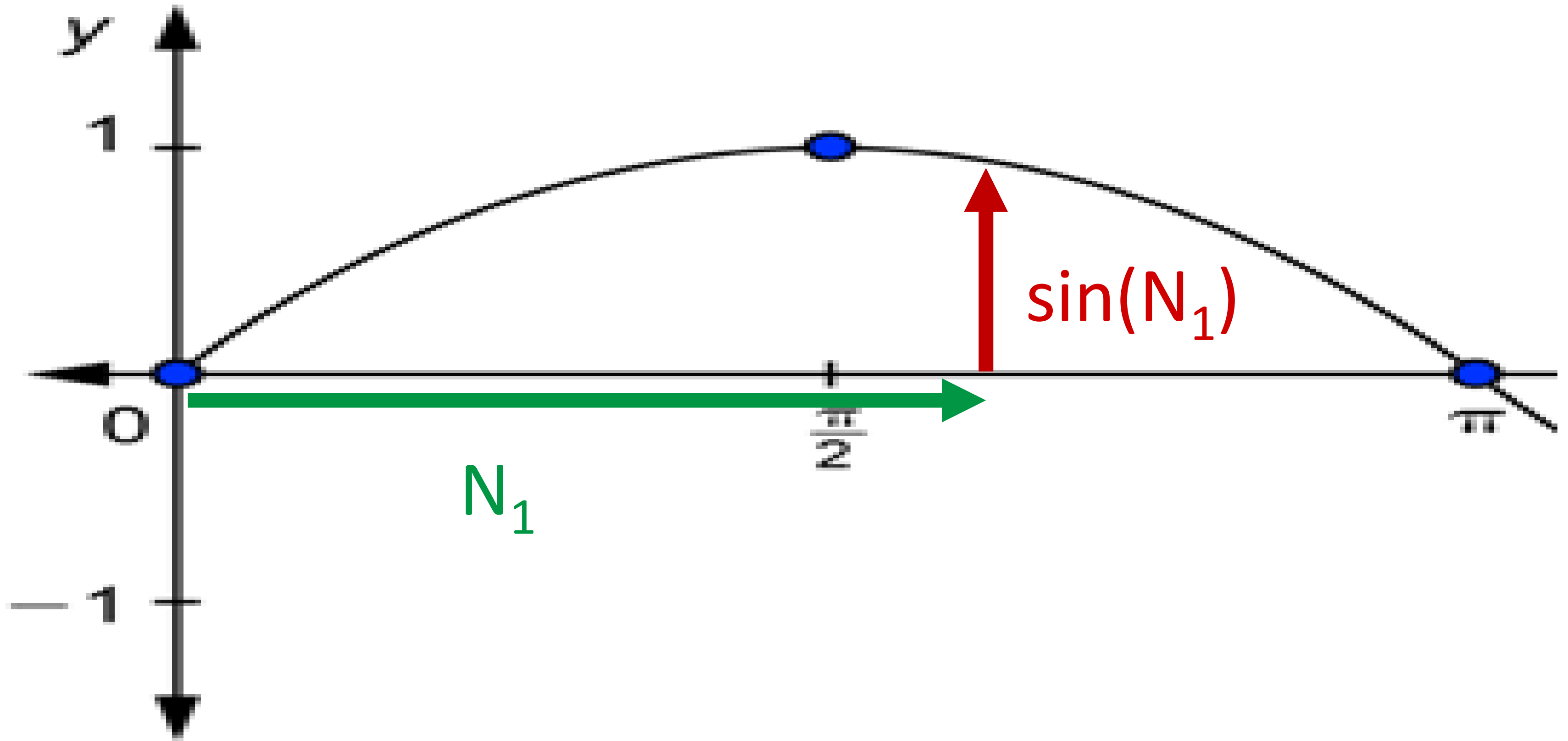



Zero at $\pi$ and it is continuous.

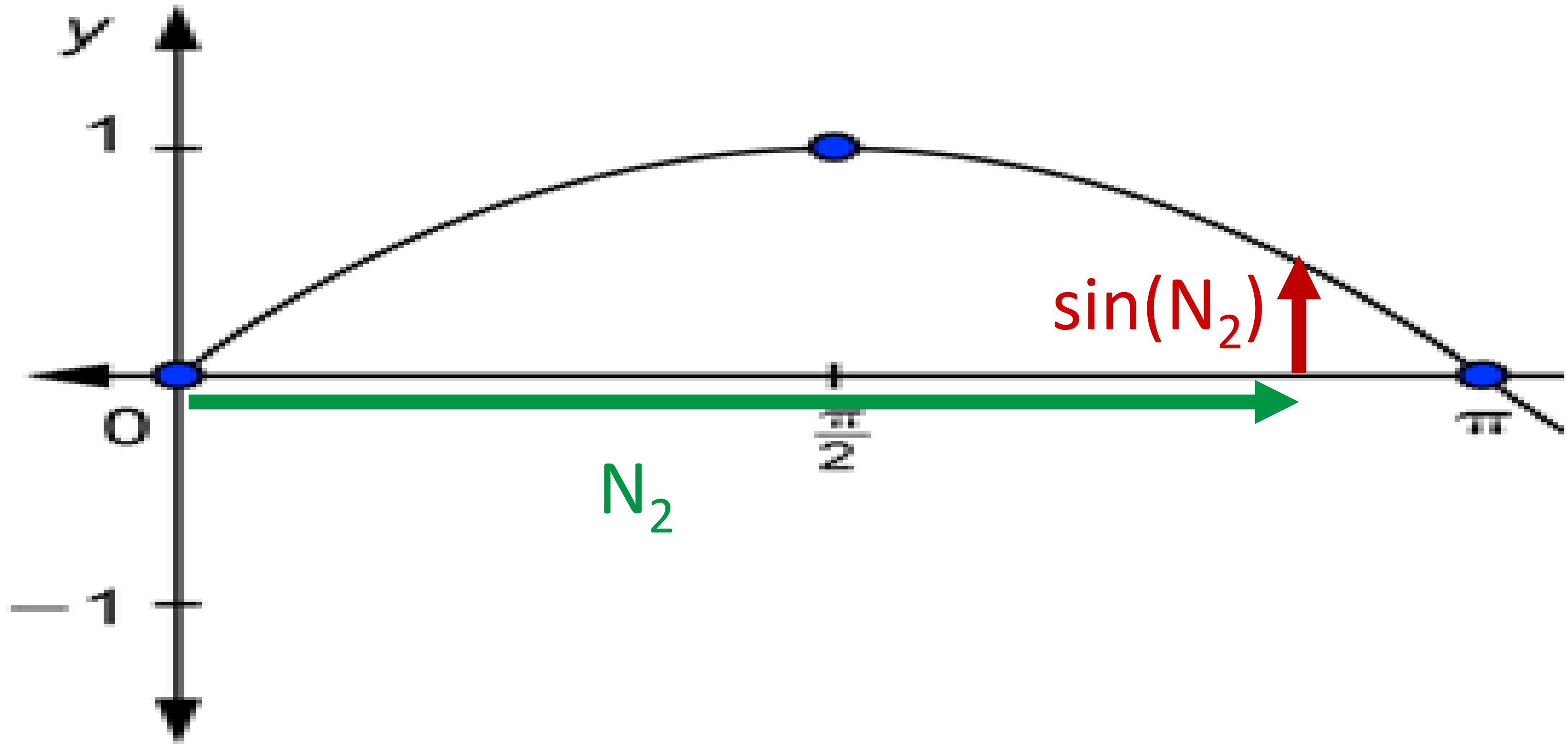
# A Geometric Example



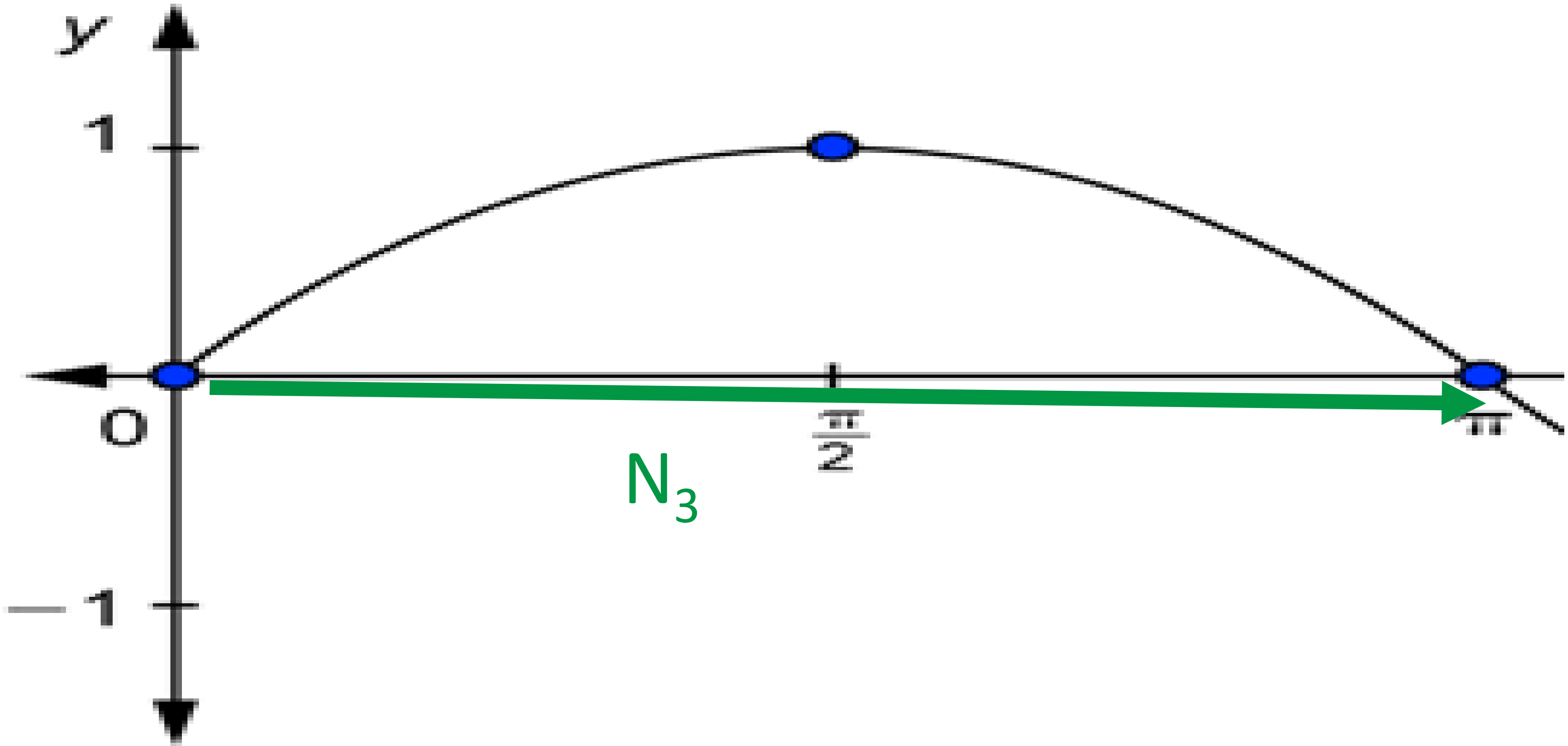
# A Geometric Example



# A Geometric Example



# A Geometric Example



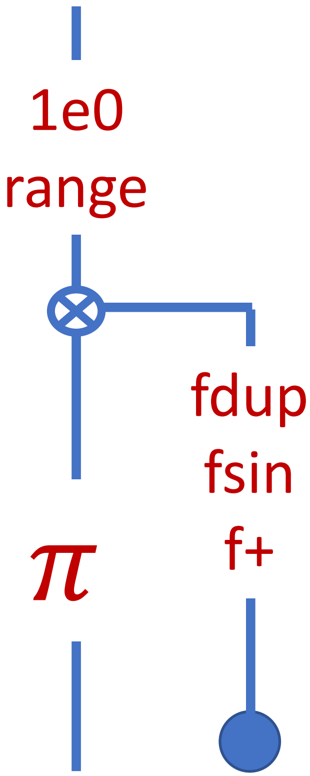
## The Logic

$$N_1 = N_0 + \sin(\mathrm{N}_0)$$

## In Forth

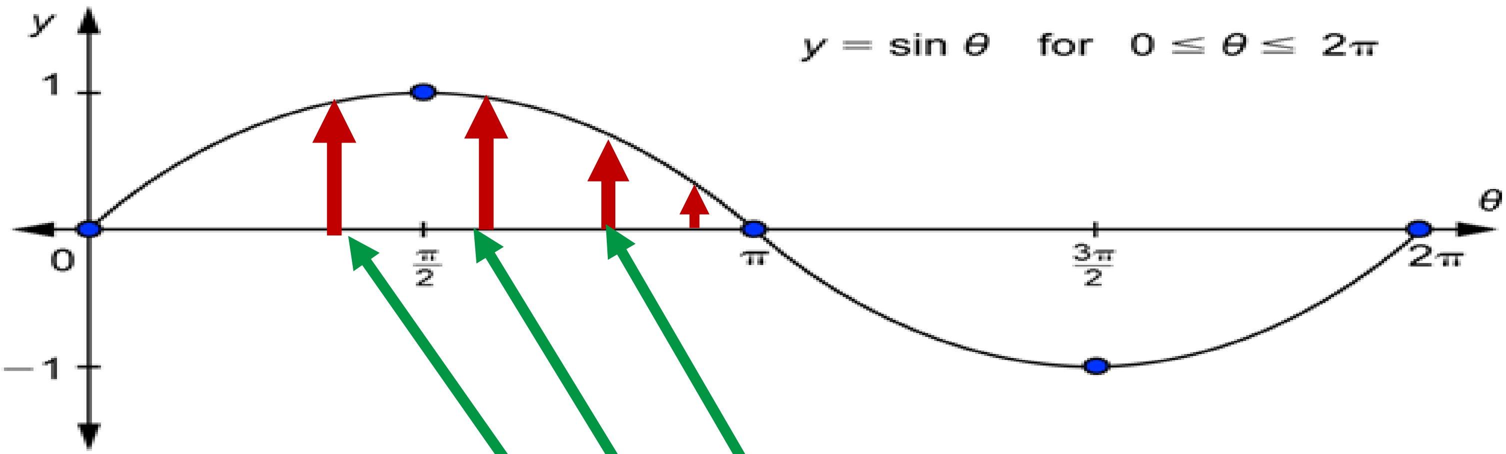
```
: one-term   fdup  fsin  f+  ;
: runpi  1e0   6  0  dup  one-term  loop ;
```

# Our Program As A D-Chart




```
: runpi  1e0  6 0  do  one-term  loop ;

: one-term  fdup  fsin  f+   ;
```




```
runpi           N_o                     sin(N_0)
1.0000000000000000   0.8414709848078964
1.8414709848078964   0.9635907245418334
2.8050617093497300   0.3302146233549986
3.1352763328997160   0.0063162786909370
3.1415926115906528   0.0000000419991400
3.1415926535897932   1.2246063538223774E-16

By sin expansion 3.1415926535897932
By cpu           3.1415926535897932  ok
```

# Summary

Yes . . . I realize any calculator or program that can compute sin can compute pi.

We are interested in exploring short cuts or clever techniques.

Plus this method gives more clarity than an infinite number series.